Critical Transport Velocity in Two-Phase, Horizontal Pipe Flow

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The suspension of solid particles or entrainment of liquid droplets in two-phase flow has for the most part been treated as a separate, unrelated phenomenon. Theoretical and empirical relationships have been derived for both instances without any consideration to the similarities between the two. However, a general relation for two-phase flow is desirable since there are systems that cannot be readily defined due to the dual (solid/liquid) nature of the transported material, such as colloids, pulp, slurries, and sludge.

Using turbulence theory, one general equation can be derived to predict critical transport velocities for two-phase horizontal flow. Simple turbulence theory was used by Davies (1987) to show the theoretical basis for existing empirical relations for solid transport in a liquid stream. The relationship that Davies derived was in close agreement with previous studies by Durand and Condolios (1952) and Oroskar and Turian (1980). This theoretical approach by Davies should be applicable to other two-phase flow systems as well.

Theoretical Derivation

Davies proposed the following force balance on an individual particle:

$$F_e \text{ (eddy fluctuation force)} = F_s \text{ (sedimentation force)} \qquad (1)$$

with

$$F_e = \pi \, \rho_f \, \hat{v}^2 \left[\frac{d_p^2}{4}\right] \qquad (2)$$

$$F_s = \left[\frac{\pi}{6}\right] d_p^3 \, \Delta\rho g(1-c)^n \qquad (3)$$

The force due to surface tension or particle adhesion, F_t, was not accounted for by Davies originally. However, if a liquid or a very cohesive slurry of solids is being transported, F_t becomes important. F_t can be defined as follows:

$$F_t = \pi \, d_p \, \sigma \text{ (for liquids)} \qquad (4a)$$

$$F_t = 4 \, \pi \, d_p^2 \, p_a \text{ (for solids)} \qquad (4b)$$

with Eq. 1 becoming

$$F_e = F_s + F_t$$

According to Davies (1987), the turbulent eddies of concern can be taken to be those of length d_p. Thus, their velocities can be given as (Davies, 1972):

$$\hat{v}^3 = P_m d_p \qquad (5)$$

with

$$P_m = \frac{2 f v_f^3}{D} \qquad (6)$$

where P_m is the power dissipated per unit mass of fluid. Combining Eqs. 5 and 6 gives $\hat{v}$ in terms of v_f:

$$\hat{v} = \left(\frac{2 f d_p}{D}\right)^{1/3} v_f \qquad (7)$$

Using Eqs. 2 and 7, and an $(1 + ac)$ term to account for the damping effects of suspended particles, Davies (1987) derived the following (shown without Davies' substitution for f):

$$F_e = (1 + ac)^{-2} \rho_f v_f^2 \left(\frac{\pi d_p^2}{4}\right) \left(\frac{2 f d_p}{D}\right)^{2/3} \qquad (8)$$

Turbulent eddies within the transporting phase can be dampened further due to waves and other disturbances on the surface of the transported phase before complete suspension is achieved. This was not accounted for by Davies (1987) in his analysis of solid transport.

In their analysis of liquid transport in a gas stream, Tatterson et al. (1977) use the term, kd_L, to account for the effects of the liquid's wavy flow on droplet generation by the gas stream.

With the Kelvin-Helmholtz mechanism for a theoretical basis, a force balance was made over an individual wave. The pressure drop, based on v_f rather than $\hat{v}$, was then multiplied by kd_L (Tatterson assumed that d_L was proportional to d_p) to account for the surface waves. This approach should be applicable to solid particle wave motion as well. Modifying Eq. 8 and introducing this along with Eq. 3 into Eq. 1 gives (in dimensionless form):

$$We = \frac{(2)^{4/3}}{\pi} f(c)_1 \left(\frac{D}{fd_p}\right)^{2/3} \left(\frac{1}{kd_L}\right) \left[\left(\frac{\pi f(c)_2}{6}\right) Bo + 1 \right] \quad (9)$$

with $We = (d_p^2 v_f^2 \rho_f / F_t)$ and $Bo = (g \Delta\rho d_p^3 / F_t)$.
Equation 9 can also be rearranged into the following form:

$$Fr = \frac{(2)^{4/3}}{\pi} f(c)_1 \left(\frac{\Delta\rho}{\rho_f}\right) \left(\frac{d_p}{D}\right)^{1/3} \left(\frac{1}{kd_L}\right) f^{-2/3} \times \left[\left(\frac{\pi f(c)_2}{6}\right) + Bo^{-1} \right] \quad (10)$$

with $Fr = v_f^2/(gD)$

It should be noted that d_p in Eqs. 9 and 10 can rarely be defined exactly. Thus, d_p is usually defined as a distribution of values (Tatterson et al., 1977; Oroskar and Turian, 1980). For solid transport, d_p can be readily defined through the direct sampling of the transport stream. However, d_p for droplet entrainment in a gas stream is determined by the flow situation.

An important characteristic of both Eqs. 9 and 10 is that the Bond number serves as the indicator of the type of phases present in the system. If surface tension forces are much greater than gravitational forces (liquid droplets in gas stream), then $Bo = 0$ and We best characterizes the system (Eq. 9). If gravitational forces predominate (the transport of noncohesive solids), then $(1/Bo)$ goes to zero and Fr best characterizes the system (Eq. 10). However, Bo may not be readily ignored without some error in Eqs. 9 and 10 for such cases as a significant amount of particle adhesion among transported solids or if $\Delta\rho$ is nonzero and σ is relatively low in a liquid-liquid system.

Additionally, the parameters, k, d_L, and F_t, need to be defined in accordance with the transport conditions (such as the type of phases involved) under consideration.

Theory vs. Previous Work

To demonstrate the appropriateness of Eqs. 9 and 10, two limiting cases are presented where comparison to prior works can be readily made.

First limiting case: solid particulate transport

For a liquid/gas-solid-particle pipe-flow system, the effects of wave motion on the critical transport velocity has been addressed rarely by researchers. This is despite the fact that the point of change from wavy flow to slug flow forms the basis for defining the critical transport velocity (Turian and Yuan, 1977).

However, research has been done in related scientific fields, most notably in the area of beach and stream erosion. For problems involving erosion in fresh-water stream beds, Kennedy (1963) and Reynolds (1965) have characterized the transition from wave motion of the stream bed and the disappearance of the waves in the bed (particle entrainment) as a function of a Froude number based on the stream depth (roughly equivalent to pipe diameter in this article) and k:

$$Fr = \frac{1}{kD} \quad (11a)$$

$$Fr = \frac{\coth(kD)}{(kD)} \quad (11b)$$

Under normal stream conditions, the critical Fr value (and hence kD) is approximately equal to one (Morisawa, 1968). Assuming that a plane geometry and a cylindrical geometry will not differ drastically in the value of critical Fr, $(kd_L)^{-1} \cong 1$ (assuming d_L is proportional to D for this system). So, if noncohesive solids are being transported, then Bo^{-1} roughly equals zero and Eq. 10 reduces to:

$$Fr = (0.42)\, f(c)_1 f(c)_2 \left(\frac{d_p}{D}\right)^{1/3} f^{-2/3} \left(\frac{\Delta\rho}{\rho_f}\right) \quad (12)$$

Equation 12 is the dimensionless version of the relation Davies (1987) derived (shown without Davies' use of the Blasius equation for f). This relation as used by Davies gives critical velocity values on the order of 30–40% higher than the empirical relation of Oroskar and Turian (1980). A better understanding of the influence of wave motion on critical Fr in a closed-pipe system could eliminate this discrepancy.

Second limiting case: transport of liquid droplets in gas stream

For a gas/liquid-liquid droplet system, two relations have been suggested for k:

$$k = \frac{1}{m} \quad (13a)$$

$$k = \frac{\rho_f v_f^2}{\sigma} f(\theta) \quad (13b)$$

where

$$\theta = \left(\frac{\rho_s}{\rho_f}\right) \frac{\sigma^2}{\eta_s^2 v_f^2}$$

Equation 13a is for a thin liquid annular layer (Tatterson et al., 1977), and Eq. 13b for a thick liquid layer (Taylor, 1940). Substituting Eq. 13a first into Eq. 9 (assuming $Bo \cong 1$ and $d_L = d_p$), Eq. 9 reduces to (with some rearranging):

$$\left(\frac{d_p}{D}\right) \left(\frac{D v_f^2 \rho_f}{\sigma}\right)^{1/2} = (2)^{2/3} f(c)_1^{1/2} \left(\frac{D}{fd_p}\right)^{1/3} \left(\frac{m}{D}\right)^{1/2} \quad (14)$$

Tatterson et al. (1977) derived the following relation for critical velocity for liquid droplet entrainment for both vertical and horizontal flow:

$$\left(\frac{d_p}{D}\right) \left(\frac{D v_f^2 \rho_f}{\sigma}\right)^{1/2} \left(\frac{f_s}{2}\right)^{1/2} = C_1 \left(\frac{m}{D}\right)^{1/2} \left(\frac{f_s}{f_i}\right)^{1/2} \quad (15)$$

with

$$f_i^{1/2} = (2)^{1/2}\left(\frac{\ddot{v}}{v_f}\right)$$

The friction velocity, $\ddot{v}$, is approximately equal to the eddy velocity, $\hat{v}$, when the turbulent eddies of interest have sufficient lengths to be classified as "energy-containing" eddies (the eddies containing the majority of turbulent energy), according to Davies (1972). As mentioned previously, the eddies of interest can be taken to be those of length d_p. If d_p is of the order of 100 to 500 μm or longer, based on the estimates from Davies (1972) (d_p values for Tatterson et al. were of this magnitude), then it can be assumed that the eddies of interest are "energy-containing" and $\ddot{v} = \hat{v}$. Substituting $\hat{v} = \ddot{v}$ and using Eq. 7 to replace the term ($\ddot{v}/v_f$), Eq. 15 becomes:

$$\left(\frac{d_p}{D}\right)\left(\frac{Dv_f^2\rho_f}{\sigma}\right)^{1/2} = (2)^{-1/3}C_1\left(\frac{D}{fd_p}\right)^{1/3}\left(\frac{m}{D}\right)^{1/2} \quad (16)$$

There is an excellent agreement between Eqs. 14 and 16, with only the minor difference being the constants of the two equations. Tatterson found that the lefthand side of Eq. 16 was basically constant, in which case the differences between the two equations become even less significant.

If Eq. 13b is used for k instead of Eq. 13a, Eq. 14 can be rewritten as:

$$\left(\frac{d_p v_f^2 \rho_f}{\sigma}\right) = (2)^{2/3} f(c)_1^{1/2}\left(\frac{D}{fd_p}\right)^{1/3} f(\theta)^{-1/2} \quad (17)$$

For the transport of low-viscosity fluids (i.e., water), θ is large, and $f(\theta)$ tends to an asymptotic value of 2/3.

Taylor (1940, Eq. 38) arrived at the following:

$$\left(\frac{d_p v_f^2 \rho_f}{\sigma}\right) = A(2\pi)x_m \quad (18)$$

where $x_m = f(\theta)^{-1}$

There are two significant differences between Eqs. 17 and 18: the exponent for $f(\theta)$ ($-1/2$ vs. -1), and the presence of an (D/d_p) term in Eq. 17. Taylor does not explicitly state the flow geometry used in his analysis, although it appears that a planar geometry is assumed. Thus, it could be argued that the adaptation of Taylor's equation to a cylindrical geometry could involve the addition of an (D/d_p) term, thereby lessening the difference between Eqs. 17 and 18.

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Notation

A = constant nearly equal to one (Taylor, 1940)
a = correction factor for damping of turbulent eddies by particles
Bo = Bond number based on F_t: ($g\Delta\rho d_p^3/F_t$)
c = volume fraction of particles in stream
D = pipe diameter
d_L = diameter of a ligament as defined by Kelvin-Helmholtz mechanism for droplet formation
d_p = particle/droplet diameter (monodispersed particles or often distribution of values)
Fr = Froude number (v_f^2/gD) = ($\Delta\rho/\rho_f$) (d_p/D) (We/Bo)
F_e = force exerted on particle/droplet by turbulent eddy
F_s = force exerted on particle/droplet by gravity
F_t = force exerted on particle/droplet by surface tension/particle adhesion (defined in Eqs. 4a and 4b)
f = friction factor
f_i = gas-phase friction factor based on interfacial shear
$f(c)_1$ = $(1 + ac)^2$
$f(c)_2$ = $(1 - c)^n$
$f(\theta)$ = x_m^{-1} of Taylor's work (1940)
g = acceleration due to gravity
h = wavelength
k = wavenumber (corresponding to v_f) ($2\pi/h$)
m = depth of liquid/solid layer in annulus
n = exponent for hindered settling
σ = surface tension (equal to $F_t/\pi d_p$ for a sphere)
p_a = particle adhesion (equal to $F_t/4\pi d_p^2$ for a sphere)
P_m = power dissipated per unit mass of fluid
$\hat{v}$ = eddy fluctuation velocity
v_f = minimum (critical) velocity of transporting phase needed for particle suspension
$\ddot{v}$ = friction velocity
$We\sigma$ = Weber number based on surface tension ($d_p v_f^2 \rho_f/\sigma$)
We = Weber number based on F_t: ($v_f^2 \rho_f d_p^2/F_t$)
x_m = dimensionless wavelength of atomizing waves (Taylor, (1940)

Greek letters

η_s = viscosity of transported phase
θ = dimensionless group defined in Eq. 13b
π = 3.14159 . . .
ρ_f = fluid density of carrier phase
ρ_s = fluid density of transported phase
$\Delta\rho$ = density difference between the two phases

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